



Accelerating Uncertainty Quantification in Estimating Seismic Collapse Capacities using Sequential Bayesian Experimental Design

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ABSTRACT

Estimating the collapse capacity of structures subjected to dynamic earthquake loading is associated with significant uncertainties due to the variability in the ground motion intensity and structural properties. Quantification of these uncertainties requires numerous nonlinear time history simulations across a wide range of input parameter values, which can be computationally intensive, particularly for high-fidelity numerical models. To address this challenge, we propose to use a sequential Bayesian experimental design framework to reduce the computational costs associated with uncertainty quantification problems in earthquake engineering. The approach sequentially identifies the most informative simulations to conduct, so that the information gain about target engineering demand parameters is maximized. Preliminary results suggest that the proposed approach outperforms Monte Carlo sampling in evaluating the statistical distributions of engineering quantities of interest, and may offer a more efficient uncertainty quantification tool in performance-based earthquake engineering applications.

Introduction

Rigorous quantification of uncertainties in the response of structures to ground shaking is crucial for performance-based seismic design [1]. Monte Carlo (MC) sampling is conventionally used to capture the influence of variability in ground motion intensity and structural properties on the seismic performance of structures, but its intensive computational demands can limit its practical use, especially for high-fidelity models [2]. Stratified sampling methods (e.g., Latin hypercube sampling [3]) have been used to improve the sampling efficiency in seismic uncertainty quantification problems [4], [5]. In addition, surrogate model-based approaches such as response surface methods [6] and polynomial chaos expansion [7] can reduce the associated computational costs. A class of methods which combines surrogate models with adaptive sampling is increasingly used to improve the estimation of seismic structural fragilities [8], [9], [10], [11]. These methods rely on sequential sampling of uncertain structural parameters or ground motion records based on active learning techniques to reduce the number of expensive numerical simulations. For example, Sainct et al. [10] used a surrogate model-based active learning framework to iteratively select the most informative ground motions for evaluating seismic fragility curves. Ning et al. [12] integrated Gaussian process (GP) regression

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with active learning to reduce the computational cost of regional-scale bridge fragility analysis by strategically selecting both informative structural parameters and ground motions. Although active learning methods can reduce the computational burden associated with MC sampling, they generally focus on reducing uncertainty in unobserved outcomes and may lead to poor overall model uncertainty quantification [13], [14]. Another class of methods, sequential Bayesian experimental design (SBED) [15], [16], [17] addresses many of the limitations of non-Bayesian active learning and explicitly targets improving global estimation or inference of statistical models. SBED relies on the sequential evaluation of a design criterion to inform the acquisition of the most informative physical experiments or numerical simulations, which optimize the parameters of an underlying model (or a function thereof) and maximize the utility of the available computational resources.

In this study, we propose the use of SBED to accelerate the quantification of aleatory uncertainty associated with the seismic collapse capacity of structures in a performance-based design context. We start by describing the formulation of the SBED and adopted design criterion [15], [14], [17], [16]. Then, we evaluate the performance of the sequential design method in capturing the influence of randomness in structural stiffness and damping properties on an extreme limit state of a bridge column. We compare the convergence of the SBED method against the MC sampling approach conventionally adopted in this context.

Methodology

The SBED procedure has two major components: (1) a surrogate model that approximates the underlying input-to-output function; and (2) a design criterion to identify the most informative simulations for evaluating a quantity of interest (QoI) based on the surrogate model approximation [16]. We use a GP regression model to approximate the mapping between the input simulation design variables $\mathbf{x}$ and the output $y(\mathbf{x})$. In our study, the design variables are the uncertain parameters of the structural model, and the simulation output is an engineering demand parameter (e.g., maximum lateral drift ratio of the structure) under a given earthquake ground motion. The predictive distribution of $y(\mathbf{x})$ given input $\mathbf{x}$ is modeled as $y(\mathbf{x}) = f(\mathbf{x}) + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_n^2)$ is Gaussian noise that reflects the uncertainty in the function approximation, and f has a GP model prior:

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')) \quad (1)$$

where $m(\mathbf{x})$ is a constant mean and $k(\mathbf{x}, \mathbf{x}')$ is a covariance matrix defined by an additive kernel, which is a combination of a Matérn ($k_{\text{Matérn}}(\mathbf{x}, \mathbf{x}'; \nu)$) and a linear kernel ($k_{\text{Lin}}(\mathbf{x}, \mathbf{x}')$):

$$k(\mathbf{x}, \mathbf{x}') = \sigma_o^2 (k_{\text{Matérn}}(\mathbf{x}, \mathbf{x}'; \nu) + k_{\text{Lin}}(\mathbf{x}, \mathbf{x}')) \quad (2)$$

where σ_o^2 is the output scale and ν is a smoothness parameter [19]. The design of the kernel was informed by a sensitivity study whose goal was to obtain the highest coefficient of determination (R^2). Our SBED design criterion is finding the optimal design ($\mathbf{x}^*$, y^*) which will maximize our knowledge about a QoI that is a scalar functional of f or y ; for example, the probability of y exceeding a certain threshold [15]. The information gained from a candidate design $\mathbf{x}$ is represented by the change in our state of knowledge about the QoI, which is quantified by the Kullback-Leibler (KL) divergence from the posterior distribution of the QoI after observing the corresponding $y(\mathbf{x})$ denoted by “ p ” to its prior distribution denoted by “ q ” [18]. To evaluate the posterior distribution of the QoI, we perform a rank-one update of the GP model conditioned on the hypothetical observation $(\mathbf{x}, y)$. The prior and posterior QoI densities are approximated by Gaussian distributions, which leads to a closed form expression for the KL divergence between the two distributions:

$$I_{KL}(\mathbf{x}, y) = D_{KL}(p \parallel q) = \frac{1}{2} \left[\log \left(\frac{\sigma_q^2}{\sigma_p^2} \right) + \frac{\sigma_p^2}{\sigma_q^2} + \frac{(\mu_p - \mu_q)^2}{\sigma_q^2} - 1 \right] \quad (3)$$

where μ_q and σ_q^2 are the mean and variance of the prior QoI distribution, and μ_p and σ_p^2 are the mean and

variance of the posterior QoI distribution after incorporating a new candidate observation $(\mathbf{x}, y)$. We calculate the expected information gain (EIG) by averaging over the random model output y :

$$I_{EIG}(\mathbf{x}) = \mathbb{E}[I_{KL}(\mathbf{x}, y^{(j)})] \approx \frac{1}{r} \sum_{j=1}^r I_{KL}(\mathbf{x}, y^{(j)}) \quad (4)$$

where r is the number of random samples of y . We select the most informative simulation by finding $\mathbf{x}^*$ which maximizes $I_{EIG}(\mathbf{x})$. A flowchart of the SBED procedure is shown in Figure 1. The GP model is trained on an initial dataset of simulations, and the corresponding prior distribution of the QoI is evaluated. The EIG in the QoI from each available candidate design $\mathbf{x}$ is evaluated, and the candidate with the maximum EIG is selected. Next, the selected simulation is performed, and the dataset is augmented with the new observation. The procedure is repeated till a decision threshold or designated number of design steps is reached.

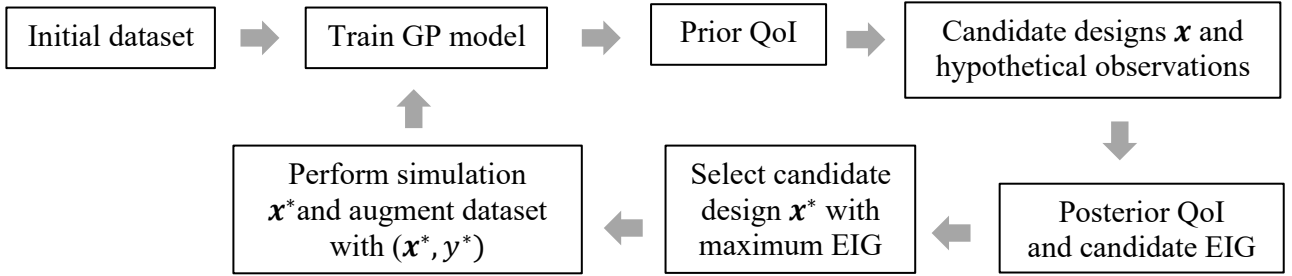


Figure 1. Flowchart of the sequential Bayesian experimental design procedure.

Example Application

A numerical model of a bridge column [20] was created using the structural analysis platform OpenSees [21] and the nonlocal fiber-section distributed-plasticity beam-column model [22], [23]. In this test problem, we consider the following sources of randomness: the compressive strength of concrete, yield strength of steel, and viscous damping ratio. All input variables were assumed to have normal distributions [24], [25], and the specimen properties reported in Schoettler et al. [20] were used to represent the mean values. The coefficients of variation of the material properties were obtained from Segura Jr et al. [24], which were based on an extensive database of laboratory tests on reinforcing bars and concrete cylinders, and the coefficient of variation for the damping ratio was obtained from Yan et al. [25].

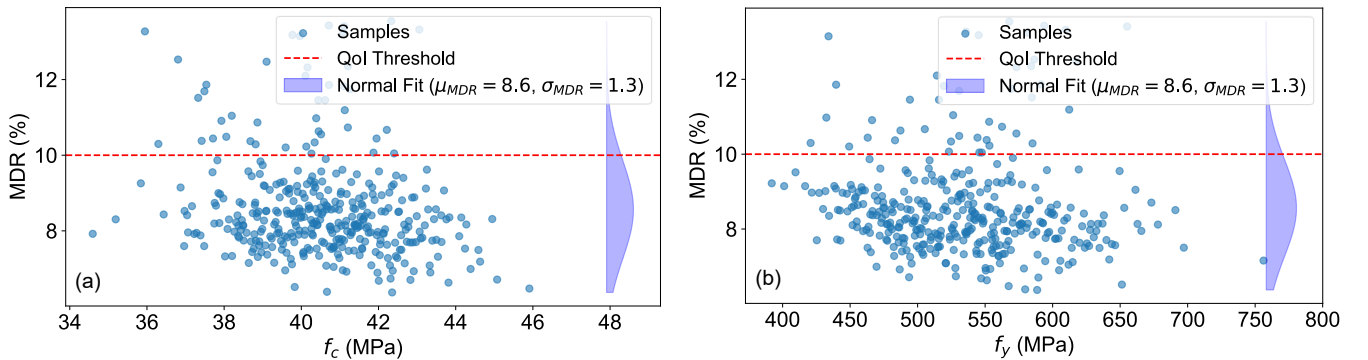


Figure 2. MC samples displaying the variability in the maximum drift ratio verses: (a) concrete compressive strength; and (b) yield strength of steel reinforcement.

The SBED procedure was used to identify the optimal set of informative simulations in a sequential manner, starting with an initial sample of eight simulations selected using a Sobol sequence method [26], and sequentially identifying the next most informative simulations in ten design steps. The EIG is evaluated on

200 randomly sampled candidate designs at each step. We subject the column to a single ground motion record (1979 Imperial Valley Earthquake) scaled by a factor of 2.72 in a nonlinear dynamic simulation and optimize the evaluation of two QoIs: the standard deviation of the maximum lateral drift ratio (MDR), and the probability of MDR exceeding 10%. This value was selected based on incremental dynamic analysis results which suggest incipient collapse near a MDR of 10%. We track the evolution of the QoIs after each design step and compare the QoI estimated by SBED against the QoI estimated with the same number of simulations selected based on MC sampling. To benchmark the performance of both methods, an empirical “true” QoI was evaluated using 500 MC samples using the NHERI SimCenter application quoFEM (Quantified Uncertainty with Optimization for the Finite Element Method) [27]. One-dimensional slices of the benchmark simulations are shown in Figure 2, and display the variation of the MDR plotted against the concrete compressive strength f_c and the yield strength of steel rebar f_y . The QoIs are also indicated in the figure: exceedance of 10% MDR, and the standard deviation σ_{MDR} of a normal distribution fitted to the MDR values.

The distributions of the QoIs estimated using each method are shown in Figure 3, where σ is the standard deviation of the QoI distribution. The QoI in subplot (a) is the standard deviation of the MDR, and the QoI in subplot (b) is the probability of MDR exceeding a value of 10%. The ground-truth QoI values are also marked in the figure. The reported QoI evolution is averaged over five independent seeds of initial samples, where the standard deviations are averaged using the law of total variance [28]. The figure demonstrates that the QoIs estimated by SBED converge toward their ground-truth values at a faster rate than the MC method. In addition, the dispersion in the QoI distributions obtained from SBED (represented by σ) is narrower than that associated with the MC method. The performance of each method is quantified by the z-score at the last design step, which is the difference between the predicted and true QoI mean, normalized by the predicted standard deviation. The z-scores for σ_{MDR} are 0.43 for SBED and 1.04 for MC sampling. The SBED-estimated probability of exceeding 10% MDR has a z-score of 0.4, and the corresponding MC-estimated QoI has a z-score of 1.32. These preliminary results suggest that SBED could reduce the computational costs associated with uncertainty quantification in performance-based design applications, compared to MC sampling.

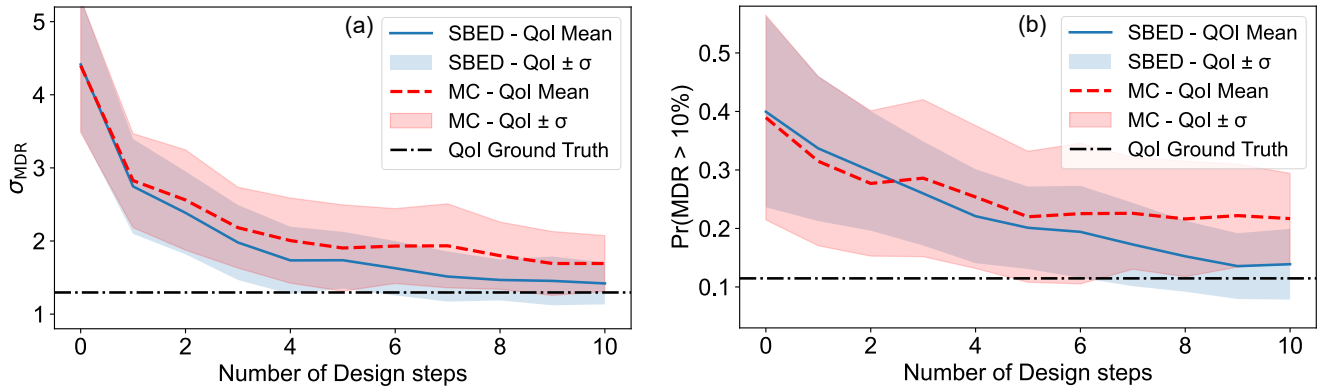


Figure 3. Evolution of the distributions of the following QoIs at multiple design steps using SBED and MC sampling: (a) standard deviation of MDR; and (b) probability of MDR exceeding 10%.

Conclusions

We propose the use of a sequential Bayesian experimental design method to select the most informative structural simulations for accelerating the quantification of uncertainties associated with extreme limit states in structures. The preliminary results of this study suggest that SBED can reduce the computational costs associated with conventional MC sampling, and may offer a more efficient tool for uncertainty quantification in performance-based earthquake engineering. Future work will test the performance of the SBED method in evaluating the influence of ground motion variability, and study its sensitivity to the input design variables, EIG estimation parameters, and surrogate model approximations.

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